Time-Dependent Friction and the Mechanics of Stick-Slip

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Abstract – Time-dependent increase of static friction is characteristic of rock friction under a variety of experimental circumstances. Data presented here show an analogous velocity-dependent effect. A theory of friction is proposed that establishes a common basis for static and sliding friction. Creep at points of contact causes increases in friction that are proportional to the logarithm of the time that the population of points of contact exist. For static friction that time is the time of stationary contact. For sliding friction the time of contact is determined by the critical displacement required to change the population of contacts and the slip velocity. An analysis of a one-dimensional spring and slider system shows that experimental observations establishing the transition from stable sliding to stick-slip to be a function of normal stress, stiffness and surface finish are a consequence of time-dependent friction.

Key words: Stick-slip; Friction.

Introduction

At the most general level laboratory observations of rock friction give reasonably consistent results. To a first approximation rocks like metals and most other materials obey two fundamental empirical laws. First, the frictional force is linearly proportional to the force acting perpendicular to the slip surface. Second, the frictional force is independent of the area of the sliding surface. Together these observations provide the conventional relationship between shear stress, τ , and normal stress, σ , acting on the surface at the time of slip:

$$\tau = \mu \, \sigma \tag{1}$$

where μ , the coefficient of friction, is assumed to be constant. Additionally, for rocks and most other materials μ is generally insensitive to composition and hardness and has values between 0.5 and 1.0. On these points most experimentalists would agree.

However, more detailed observations reveal a variety of interesting phenomena and relationships that show μ is not constant. Frequently, a given set of observations may prove to be elusive, defies intuitive explanation and is often found to apparently contradict some other set of experimental observations. This situation implies, of course, that unrecognized variables are affecting the results. Much of the recent published work on rock friction has dealt with the variables that may affect the

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details of frictional response. With the recent increased interest in the mechanics of earthquake faulting as they bear on earthquake prediction, several questions related to the cause of unstable fault slip (stick-slip) are of particular interest. Some topics of interest that are discussed here include: the relationship between stick-slip and stable sliding; the relationship between static and sliding friction; and the role of preseismic slip to the stick-slip instability. It is the thesis of this paper that at least partial solution to the above problems is found in fairly subtle interactions between time-, velocity- and displacement-dependence of μ and the combined elastic characteristics of the test machine and sample system.

Time-dependency

Time-dependency of static friction of rocks was first noted by DIETERICH (1970, 1972). In those experiments slip on surfaces of granite, graywacke, quartzite and sandstone showed an increase of the coefficient of friction, μ , with the logarithm of the time of stationary contact (Fig. 1). For these experiments τ and σ were held constant for intervals as long as 10⁵ seconds. At the end of the interval, the shear



Figure 1 Time-dependence of the coefficient of static friction of quartz sandstone (from DIETERICH, 1972).

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stress was then rapidly increased to the critical level required to produce slip. The results satisfy the empirical law:

$$\mu = \mu_0 + A \log (Bt + 1) \tag{2}$$

where t is the time of contact, and A, B and μ_0 are constants. Note that relationship (2) is not the form originally employed to represent the data in DIETERICH (1972). The earlier form:

$$\mu = \mu_0 + A \log \left(t \right) \tag{3}$$

gives negative μ as t approaches zero. With the present relationship approaches constant value equal to μ_0 for $t < B^{-1}$. For intervals of approximately 1.0 seconds and less DIETERICH (1972) reports that μ is constant and no time-dependency is observed. Hence, for those data (DIETERICH, 1972, Table 1) *B* in equation (2) is approximately equal to 1.0 and the constants μ_0 and *A* remain as given using relationship (3). The constants μ_0 and *A* are insensitive to normal stress. Average values of *A* are 0.016, 0.022, 0.020, and 0.012 for sandstone, granite, quartzite and graywacke, respectively. Average values of μ_0 are 0.7–0.8.

Because the magnitude of the time-dependent effect is small compared to both the uncontrolled variability of μ between stick-slip events and the often observed overall increase in μ with displacement, the time-dependent effects may be easily masked. DIETERICH (1972) observed time-dependency only for rough ground surfaces that were separated by a thin layer of displacement-produced wear particles. Studies by SCHOLZ *et al.* (1972), ENGELDER *et al.* (1975), SCHOLZ and ENGELDER (1976), ENGELDER and SCHOLZ (1976) and TEUFEL (1976) give direct and indirect data that show time-dependent increase in the static coefficient of friction is indeed a general characteristic of rock friction under a variety of test conditions and slip surface properties.

It is interesting to note that apparently analogous time-dependent behavior is found under other circumstances in ceramic powders and in soil materials (sands, silts and clays). For example, the strength of dry compacted ceramic powders increases



Figure 2 Time-dependence of the dynamic shear modulus of dry quartz sand (redrawn from AFIFI and WOODS, 1971).

with the logarithm of the duration of compaction. Similarly, studies by AFIFI and WOODS (1971), MARCUSON and WAHLS (1972), TRUDEAU *et al.* (1974), and ANDERSON and RICHART (1974) establish that the dynamic shear modulus of air-dried sands, silts and clays increases with the logarithm of time during secondary compaction at constant confining pressure (Fig. 2). Hence, these observations suggest time-controlled strengthening of grain-to-grain contacts within the aggregates by mechanisms that are probably similar to those for the friction observations.

Mechanism of time-dependency

Possible mechanisms of processes causing time-dependent increases in rock friction have been discussed by DIETERICH (1972), SCHOLZ et al. (1972) and SCHOLZ and ENGELDER (1976). DIETERICH (1972) observed the similarity with time-dependent friction in metals (for examples, RABINOWICZ, 1965) which has been ascribed to one of two possible mechanisms (BOWDEN and TABOR, 1964). First, the real area of adhesive contacts between the surfaces may increase because of localized creep resulting in time-dependent reductions of near-surface hardness. Second, the actual area of adhesive contacts may remain constant with time, but the strength of the contacts increases because of time-dependent breakdown of surface films that interfere with adhesion. For rock friction DIETERICH (1972) noted that time-dependent reductions of microhardness are seen for at least some non-metallic substances (WALKER and DEMER, 1964) and hence are suggestive of the first mechanism, but the evidence did not seem sufficiently strong to eliminate the possibility of the second mechanism. Similarly, SCHOLZ et al. (1972) suggested increase of contact area by creep at points where the surfaces touch as the mechanism for time-dependent friction.

Overall, the evidence now appears to strongly favor creep at points of contact or equivalently, time-dependent reductions of near surface hardness as the cause of time-dependent friction. The actual mechanism may consist of increases in the real area of adhesive contact or perhaps, as suggested by SCHOLZ and ENGELDER (1976) and ENGELDER and SCHOLZ (1976), by increased asperity penetration and ploughing. Several authors (for example, BowDEN and TABOR, 1964; RABINOWICZ, 1965; JAEGER and COOK, 1969) discuss the contribution to the friction force that may be made by ploughing of an asperity on one surface through the material on the other surface. For metals, and presumably for rocks, the magnitude of this force is generally considered negligible compared to adhesion. Two situations where ploughing may be important are: 1) penetration of a surface by a harder angular asperity, and 2) in situations where a film of lubricant or surface contamination (i.e. boundary lubrication) reduces the contribution to the friction force by adhesion. For both adhesion and asperity ploughing, time-dependent reductions of surface hardness by creep lead to similar time-dependent increases in friction. In addition to the data of WALKER and DEMER (1964) noted above, WESTBROOK and JORGENSEN (1965, 1968) and SCHOLZ and ENGELDER (1976) and several other studies show that indentation creep which gives a time-dependent increase in the area of contact is characteristic of many non-metals including several minerals (Fig. 3). WESTBROOK and JORGENSEN show that the observed reduction of surface



Figure 3

Time-dependence of the area of indentation of olivine and quartz (from SCHOLZ and ENGELDER, 1976).

hardness by creep of the minerals corundum, periclase, quartz, calcite and fluorite under atmospheric conditions is caused by adsorbed water. They demonstrate that the softening effect is reduced or eliminated when the specimens are tested in a waterfree environment following heating to 300°C in an argon atmosphere to remove adsorbed water. WESTWOOD et al. (1967) have specifically identified the indentation creep effect as belonging to a larger class of chemically induced changes in surface hardness by surface-active species (WESTWOOD et al. 1967). Studies by WESTWOOD et al. (1967), WESTWOOD and GOLDHEIM (1968) and MACMILLAN et al. (1974) have shown that surface microhardness can also be controlled by other adsorbed ions and molecules. Those studies establish that in chemically active environments surface hardness is at a maximum when the zeta-potential of the surface is near zero. Zetapotential is the electrical potential at the boundary between the inner adsorbed layer and the other diffuse layer of ions. Zeta-potential may be controlled by highly charged complex ions, or organic molecules possessing a high dipole moment. Changes of surface mechanical properties are caused by changes in the near-surface dislocation mobility (WESTWOOD et al., 1967; WESTWOOD and GOLDHEIM, 1968; and MACMILLAN et al. 1974). For example, studies of indentation creep of MgO which is amenable to dislocation etch-pitting show that time-dependent dislocation motion is reduced or eliminated in a water-free toluene environment and that dislocation motion during indentation in aqueous environments and various solvent environments is highly timedependent (WESTWOOD *et al.* 1967; WESTWOOD *et al.* 1968). Additionally, it is found that the rates of dislocation motion are extremely sensitive to the ionic composition of the aqueous environment (WESTWOOD *et al.* 1967) and that rates are minimized in environments giving zero zeta-potential (MACMILLAN *et al.* 1973). Hence, the indentation creep effect in a chemically active environment can be minimized at zero zeta-potential.

MACMILLAN et al. (1974) propose that the well-known increase of friction between surfaces of glass in the presence of water is caused by chemically induced changes in surface microhardness, not as a disruption of surface contaminates as proposed by HORN and DEERE (1962), nor by surface tension of water as proposed by BYERLEE (1967). MACMILLAN et al. further argue that such effects may be widely important in determining friction in non-metals. At this juncture the reader will recall the friction mechanisms dominated by adhesion or ploughing predict *increased* friction with indentation creep because the area of contact and/or depth of asperity penetration are increased which in turn increases the strength of the contacts between the surfaces. Conversely, the brittle asperity of BYERLEE (1967) predicts that asperity creep will result in the failure of interlocked asperities at reduced stresses giving a lowered friction. MACMILLAN et al. (1971) find that both MgO and soda-lime glass give minimum friction when the zeta-potential was zero (i.e. when the indentation creep effect is minimized). This result suggests that adhesion and/or asperity ploughing are the dominant friction processes. The results of MACMILLAN et al. (1974), which were obtained using a variety of organic fluids, water and buffered 10^{-2} N NaCl solutions to control zeta-potential show that if boundary lubrication effects are eliminated, a minimum in friction can be obtained in any environment with surfaceactive species that produces a zero zeta-potential because it maximized the microhardness. MACMILLAN et al. note the interesting observation that n-hexadecane, which is an oily, viscous fluid, is associated with higher dislocation mobilities in MgO than is distilled water and produces a higher frictional resistance than a distilled water environment.

Systematic study of the effect of chemically induced hardness changes for rock friction are limited to a brief study by SWOLFS (1971). SWOLFS examined slip on sawcuts of Coconino Sandstone in the presence of 10^{-3} N AlCl₃ solutions which produce surface weakening of quartz compared to dry conditions or tests with pure water. The AlCl₃ solution produced an 8 percent increase in the coefficient of friction. This effect is in agreement with the monomineralic tests by MACMILLAN *et al.* (1974) and may be explained by increased area of adhesive contact.

In summary, the work on adsorption of surface-active species establishes that the dominant mechanism of friction is by asperity adhesion and/or ploughing and that the time-dependence of friction is related to creep at points of contact that increases the area of adhesion or depth of penetration.

A final item of evidence demonstrating increase of area of adhesive contacts by

asperity creep as a mechanism of time-dependent friction is given by TEUFEL (1976) and TEUFEL and LOGAN (1977). TEUFEL conducted a series of friction experiments on sawcuts of Tennessee Sandstone at displacement rates from 10^{-3} cm/sec to 10^{-7} cm/sec. Stick-slip was observed at all rates of shortening except 10^{-3} cm/sec at which the sliding was stable. With decreasing rates of displacement and hence increasing duration of contact between stick-slip events, the friction increased. The use of thermo-dyes that change color in response to slip induced temperature changes at points of contact, permitted measurement of the actual area of contact. The results show that contact area increases with increasing time of contact between stick-slip events. Additionally, as might be expected, the average normal stress computed for the contact points decreases as time increases.

Velocity-dependent friction

It is often assumed that sliding friction of rocks is independent of slip velocity. However, the time-dependence of static friction suggests the possibility of velocitydependent friction. Displacement-induced wear can change the coefficient of friction during the course of an experiment and has therefore tended to make systematic experiments for velocity-dependence difficult. Recently the velocity effect has been demonstrated. SCHOLZ and ENGELDER (1976) observe velocity-dependency of μ for slip of corundum on Westerly granite, corundum on Solenhofen limestone, and corundum on Twin Sisters dunite. They observe that μ is inversely proportional to the logarithm of slip velocity from ~10⁻⁴ mm/sec to ~10⁻¹ mm/sec.

Figure 4 gives data which was orally presented at the December 1975 meeting of the AGU for velocity-dependent friction for slip between surfaces of Westerly granite at 19.6 bars normal stress. These experiments were conducted using the 'sandwich' type direct shear configuration described by DIETERICH (1972). This



Figure 4 Velocity-dependent friction. A, B and C refer to different experimental runs. The bars give the observed variation of μ for slip at constant velocity.

arrangement permits the displacements, which take place at constant normal stress, to exceed 1 cm before it is necessary to reset the sliding block. These slip surfaces were ground flat using #240 abrasive. Prior to making velocity measurements the block was displaced and reset several times for a total displacement of approximately 5 cm. The purpose of this procedure was to produce a thin layer of wear-generated gouge that is found to reduce additional displacement dependence on μ . To measure μ , velocity was held constant for sufficient time to give at least 1 mm of displacement. After 1 cm of displacement it was necessary to terminate an experimental run in order to reset the sliding block. In Fig. 4, A, B, and C refer to three different experimental runs. The bars give the variation of μ at a particular velocity. It was found that repositioning the slider could cause an apparent change of the overall coefficient of friction. For purposes of comparison, the data in the figure were positioned to show similar μ at 10^{-2} cm/sec.

During the course of these experiments, it was found that when the velocity of slip was abruptly changed, the frictional resistance did not change immediately. The force-displacement records (Fig. 5) show that a critical displacement, d_c , is required before the friction stabilizes at a value characteristic of the new slip velocity. The



Figure 5 Variation of frictional stress as a function of displacement for change of slip velocity.

parameter d_c was measured under a variety of test conditions to determine if it could possibly be a machine effect or if it is controlled by experimental conditions such as velocity or stress. No such dependence was found. Even at the lower range of velocities $(5 \times 10^{-4} \text{ cm/sec} \text{ to } 10^{-6} \text{ cm/sec})$ the response of the apparatus is rapid and a change of velocity is recorded as a smooth uninterrupted change of sample shortening velocity that requires less than 0.5 seconds to take place. At those rates this transition time of < 0.5 seconds is very much less than the sliding time required to give a displacement equal to d_c . Hence d_c was measured at essentially constant velocity and could not arise as a consequence of some machine effect such as the taking up of slack when the speed was changed. Within the velocity range where d_c could be accurately measured $(0.5 \times 10^{-4} \text{ cm/sec} \text{ to } 10^{-6} \text{ cm/sec})$ and to 450 bars normal stress, no velocity or stress dependence was found. However, d_c was found to be sensitive to surface roughness. The parameter d_c appears to be an intrinsic quality of the surfaces. If the interactions between the surfaces at points of contact during slip are viewed as a statistical James H. Dieterich

process in which the population of contacts continuously changes in response to changes of displacement and velocity, then d_c , may be interpreted to represent the displacement required to eliminate the population of contacts characteristic of the previous velocity. Therefore d_c would be related to the average dimensions of the zones of contact or perhaps to some average shear strain, γ , of the contact zone that is required to break contacting asperities:

$$\gamma = \frac{d_c}{h} \tag{4}$$

In (4) h is the effective width of the contact zone and is presumably proportional to surface roughness or to gouge thickness. Estimates of h give values for γ of 0.2 to 0.5.

These observations suggest a model for sliding friction in which the resistance to slip results from two competing processes. First, from the observation that friction increases with time of stationary contact and from the indentation creep data it is asserted that the points of contact between asperities tend to become stronger with age because of creep-induced increases in area of contact (i.e. equation (2)). Second, displacement causes the destruction of old and hence strong points of contact which are then replaced with new and consequently weaker points. This model then implies a velocity-dependence of friction since the effective lifetime, T, of a point of contact is inversely proportional to slip velocity, V:

$$T = \frac{d_c}{V} = \frac{\gamma h}{V} \tag{5}$$

Hence, if t, the time of stationary contact in equation (2) is replaced with T, the average lifetime of a population of contacts at a steady velocity:

$$\mu = \mu_0 + A \log\left(\frac{Bd_c}{V} + 1\right) \tag{6}$$

or

$$\mu = \mu_0 + A \log\left(\frac{B\gamma h}{V} + 1\right) \tag{7}$$

Equations (6) and (7) have a logarithmic dependence on velocity of the form indicated by the data in Fig. 4 and the SCHOLZ and ENGELDER (1976) data for slip of corundum on various rock surfaces. The apparent success of this model is illustrated by the solid curve through the data in Fig. 4. The curve gives a fit to the data using equation (6). For this fit the value μ_0 is taken to be the observed value of μ at $V = 10^{-2}$ cm/sec. The constants A and B have values of 0.02 and 1.0 and were obtained from the static friction data for granite in DIETERICH (1972). The critical displacement, d_c , has a value of 5×10^{-4} cm and was obtained directly from the force-displacement data for the experiments.

Mechanics of stick-slip

RABINOWICZ (1965) proposes a model for stick-slip based on observations of time-dependent friction in metals. The essential characteristics of the model are that: static friction increases with the logarithm of the duration of contact; sliding friction is a constant; and for very small times of contact static friction equals sliding friction. This model has two interesting implications for stick-slip with time-dependent rock friction that have been noted by DIETERICH (1972) and SCHOLZ *et al.* (1972). First, for very short times of contact, static and sliding friction are equal and slip is stable. Second, for finite contact times, static friction exceeds sliding friction and stick-slip instability occurs. The stress drop during stick-slip is proportional to the logarithm of the time of contact. Observations by DIETERICH (1972), SCHOLZ *et al.* (1972), ENGELDER *et al.* (1975), SCHOLZ and ENGELDER (1976), TEUFEL (1976) and TEUFEL and LOGAN (1977) conform to those aspects of the model. However, this model does not account for several other characteristics of stable and unstable slip in rocks.

The purpose of the following discussion is to show the implications of velocity dependent friction, equations (6) and (7), and critical displacement, d_c , on the mechanics of stick-slip and stable sliding. Observations of velocity-dependent friction and critical displacement together with time-dependent static friction unify and provide an explanation for three basic and previously puzzling characteristics of rock friction phenomenology. Those characteristics are: 1) dependence of the transition of between stick-slip and stable sliding on normal stress. It is widely reported that stable slip at low normal stress can become unstable (stick-slip) at higher normal stress. Some theories of friction and stick-slip have placed special significance on the particular minimum normal stress at which stick-slip has been observed to take place (BYERLEE, 1970; ENGELDER and SCHOLZ, 1976). However, there has been no agreement in the literature on the normal stress at which this occurs. For example, with Westerly granite, SCHOLZ et al. (1972) report a minimum normal stress of approximately 10 bars while BYERLEE and BRACE (1968) report a normal stress of 1.2 kilobars. 2) Dependence of stick-slip on stiffness of the test system. Stiffness is variously defined as either the change of force or the change of stress divided by displacement. For metallic friction, stiffness is widely recognized and discussed as a determinant of stick-slip. For rock friction, the role of stiffness has received less attention, but OHNAKA (1973) reports that increased machine stiffness decreases the tendency for stick-slip as observed in metals. Similarly, comparison of published experimental data indicates that low machine stiffness enhances the tendency for stick-slip at low normal stresses. For example, with the above noted data on the normal stress required to produce stick-slip in Westerly granite, the apparatus used by SCHOLZ et al. (1972) has a vertical stiffness of $\sim 2.3 \times 10^4$ kg/cm (obtained from SCHOLZ et al. (1972) Fig. 5 at the transition from stable sliding to stick-slip) while the apparatus used by BYERLEE and BRACE (1968) has a stiffness of 20×10^4 kg/cm

(BYERLEE, personal communication). Because those studies used different size samples the stiffnesses show an even greater contrast if expressed in terms of change of stress rather than change of force. Dividing the above values by the end area of the samples yields 0.74 kb/cm and 99 kb/cm for the SCHOLZ et al. (1972) and BYERLEE and BRACE (1968) data respectively. This two-orders-of-magnitude contrast in stiffness (expressed as stress/displacement) is possibly related to the two-orders-of-magnitude difference in the minimum normal stress for stick-slip found for the two experiments. New data presented below show that there is indeed a direct correspondence between stiffness and normal stress required to produce stick-slip. 3) Surface finish effects. Several authors have observed that surface finish and the presence of gouge have some control on the stability of slip (HORN and DEERE, 1962; BYERLEE, 1967; HOSKINS et al., 1968; JAEGER and ROSENGREN, 1969; DIETERICH, 1972; SCHOLZ et al., 1972; OHNAKA, 1973). While there is no unanimity of opinion, the data for the most part suggest that the greater the surface roughness, the lesser the tendency for stick-slip. The presence of gouge on the slip surface appears to reduce the tendency for stick-slip in some situations (SCHOLZ et al., 1972) with thicker layers of gouge having less tendency for stick-slip than thin layers (BYERLEE and SUMMERS, 1976).

Figure 6 presents the results from a series of experiments for the transition from stable sliding to stick-slip as a function of normal stress, stiffness and surface finish. The direct shear arrangement described above was used with Westerly granite that had surfaces finished with #600 and #240 abrasives. Stiffness was varied by placing various elements having different stiffnesses between the specimen and the loading ram that supplies shear stress to the surface. The experimental procedure was to incrementally increase the normal stress until stable slip changed to stick-slip. At a constant normal stress, the time of stationary contact of the block was fixed at 60 seconds. At the end of that interval, the shear stress was then rapidly increased to produce slip. If slip was stable, then the normal stress was increased and the block was again held in contact for 60 seconds. The procedure was repeated until stick-slip occurred. The data points in Fig. 6 record the minimum normal stress and stiffness for stick-slip. Stiffness, K, was measured from the displacement, d, and stress drop, $\Delta \tau$, for the stick-slip event:

$$K = \Delta \tau / d$$

Figure 6 demonstrates that the normal stress at the transition from stable sliding to stick-slip is approximately linearly proportional to stiffness. At the same stiffness, stick-slip occurs at lower normal stress for finely ground surfaces than it does for the rougher surface. For each surface stick-slip could be induced at a few bars normal stress, if the stiffness was sufficiently low. It is noted that the SCHOLZ *et al.* (1972) and the BYERLEE and BRACE (1968) data for stiffness and normal stress described earlier plot close to the straight line passing through the data for the #240 surface in Fig. 6.

It is further noted that the results in Fig. 6 and the finding from DIETERICH (1972) that the increase of μ with time is independent of normal stress are both in apparent



Figure 6 Transition from stable sliding to stick-slip as a function of normal stress, stiffness and surface finish. The data points give the minimum normal stress required to give stick-slip.

conflict with the conclusions of ENGELDER and SCHOLZ (1976) for the mechanism of stick-slip. Based on microscopic examination of wear during stick-slip and stable sliding (see also ENGELDER, 1974) they note that stick-slip is associated with wear grooves and micro-cracking of the surface while stable sliding is not. They conclude that time-dependent stick-slip occurs only if the normal load is sufficiently large to cause cracking during static contact, and that the normal stress at the stable sliding to stick-slip transition corresponds to the minimum normal stress to cause asperity indentation and ploughing. However, the above results show that stick-slip can occur at any normal stress if the stiffness is sufficiently low and that time-dependent changes in μ are apparently independent of normal stress at least over the range 20–850 bars. It would appear therefore that stick-slip does not arise as a consequence of microcracking, but rather that micro-cracking and wear grooves occur as a consequence of stick-slip. Given the demonstrated time-dependent mechanical properties of points of contact between sliding surfaces it would not be surprising if different modes of deformation (i.e. micro-cracking and grooving) came into play at the high slip velocities characteristic of unstable slip.

Stick-slip clearly arises from an interaction of the mechanical properties of the slip surface with the sample/machine system that exerts stress on the surface. Some characteristics of stick-slip and stable sliding with time- and velocity-dependent friction may be analyzed with the conventional one-dimensional spring and slider arrangement.

Consider the interaction that will take place when the slider which has been stationary for some time, t, begins to move in response to force exerted through the spring (stiffness of K) which extends at the loading velocity V_i (Fig. 7). If the spring



Figure 7 Stable slip of a one-dimensional slider that has time- and velocity-dependent friction.

is infinitely stiff, the slider will immediately move at velocity V_l and the frictional force will drop from the value characteristic of the contact time, t, to the steady state value characteristic of V_l . For velocities up to the point at which V_l approaches the magnitude of $(B)(d_c)$, equation (6) specifies that the frictional strength will show an inverse dependence on the logarithm of V_l . The displacement over which the friction falls from the static to the sliding value assumed to be given by d_c . The exact stress-displacement path followed during the falling strength portion of the curve depends upon the details of the population of the points of contact and for the case of finite spring stiffness upon velocity-friction-spring distortion interactions. For purposes here, a linear path is used.

With this model stick-slip occurs whenever the decrease of frictional strength with displacement exceeds the characteristic unloading curve for the system. The unloading curve has a slope equal to the spring stiffness. Hence, high-stiffness systems will behave approximately like the infinite stiffness example and show stable sliding. Low-stiffness systems are unable to follow the strength-displacement path which results in unbalanced forces and a stick-slip event (Fig. 8). Because the slider has a high velocity, V_{ss} , during stick-slip, equation (6), predicts that in general sliding friction during stable slip.

The transition from stable sliding to stick-slip takes place when the slope of the unloading path, -K, equals the slope of the path followed when the friction falls from the static to the sliding value:

$$-K = -\Delta \tau / d_c \tag{8}$$



Stick-slip of a one-dimensional slider with time- and velocity-dependent friction. The solid line gives the variation of friction with displacement. The dashed line gives the force-displacement path of the spring. Unstable slip occurs because the force exerted by the spring exceeds the frictional resistance.

Where $\Delta \tau$ is the difference between the static and sliding friction. From (1):

$$\Delta \tau = \Delta \mu \sigma \tag{9}$$

and from (2) and (6)

$$\Delta \mu = A \left[\log \left(Bt + 1 \right) - \log \left(B \frac{d_c}{V_l} + 1 \right) \right]$$
(10)

Combining (8) and (9) yields

$$\sigma = K d_c / \Delta \mu \tag{11}$$

Hence, the normal stress at the transition from stable sliding to stick-slip is linearly proportional to stiffness and d_c . As noted above, d_c is apparently related to the dimensions of the contacting asperities and presumably scales by roughness. Therefore, from (4) and (11) the normal stress for the transition is also expected to be proportional to h, the roughness and perhaps gouge thickness:

$$\sigma = K\gamma h / \Delta \mu \tag{12}$$

These predictions of the model conform well to the results of Fig. 6. The solid lines in Fig. 6 represent a fit to the data using equation (11) with the experimentally determined values A = 0.02, B = 1.0 from DIETERICH (1972); time of static contact, t = 60 seconds; and loading velocity, $V_l = 10^{-2}$ cm/sec. Critical displacements, d_c , are estimated to be 5×10^{-4} cm and 1×10^{-4} cm for the #240 and #600 surfaces respectively. It is interesting to note that consistent values for the stick-slip displacement were measured at the transition, independent of stiffness or normal stress. Those values which are 1.2×10^{-3} and 2×10^{-4} cm for the #240 and the #600 surfaces respectively are approximately twice the magnitude of d_c . Reference to Fig. 8 indicates that the displacement for stick-slip at the transition should be a constant that is related to d_c . This suggests that the stick-slip displacement at the transition divided by the appropriate factor (~2) may provide a simple method of estimating d_c .

Equation (10) suggests that the normal stress for the transition from stable sliding to stick-slip is also controlled by the duration of stationary contact preceding slip and by the driving velocity. Experiments are planned to look for these possible effects.

Summary and conclusions

A theory of friction has been proposed that establishes a common basis for static and sliding friction. In the absence of time-dependent effects static and sliding friction are each equal to the same constant value, μ_0 . Increase of friction above μ_0 arises from time-dependent effects and is proportional to the logarithm of the time that a population of points of contact exists. For static friction, that time is simply the time of stationary contact. For sliding friction the time of contact is determined by the velocity of slip and the critical displacement required to change the population of contacts.

From the observations of SCHOLZ and ENGELDER (1976), TEUFEL (1976) and TEUFEL and LOGAN (1977) it appears quite likely that creep at points of contact on the surfaces causes the time dependency. The observations of WESTBROOK and JORGENSEN (1965, 1968) show that asperity creep depends upon adsorbed water. Therefore, it might be expected that time-dependent effects and stick-slip would be reduced or eliminated if experiments were conducted in a water-free environment. Those experiments have not been done.

It was demonstrated experimentally that the transition from stable sliding to stick-slip is a linear function of normal stress, stiffness and surface roughness. Analysis using a simple one-dimensional spring and slider model shows that those observations are a direct consequence of time-dependent friction.

In the present form the one-dimensional model does not predict observations of premonitory creep for stick-slip. It does not exclude it, however. Because the friction is velocity-dependent, unstable slip of an already moving block may take place by a perturbation of the slip velocity. Detailed observations of preseismic slip (DIETERICH, 1975, 1977) support that interpretation. In general, those experiments show that creep begins at some clearly indentifiable point on the slip surface and propagates slowly across part or all of the sample. Hence, part or in many cases all of the slip surface was slowly creeping prior to the time of seismic slip and rapid stress drop. Those experiments establish in addition that preseismic slip arises from heterogeneity of the frictional strength measured relative to the initial shear stress along the surface. It was found that if the heterogeneity was sufficiently reduced, preseismic slip did not occur.

References

- AFIFI, S. S. and WOODS, R. D. (1971), Long-term pressure effects on shear modulus of soils, J. Soil Mech. and Foundations Div., ASCE, 97, No. SM10, 1445–1460.
- ANDERSON, D. G. and RICHART, F. E. Jr. (1974), Temperature effect on shear wave velocity in clays, J. Geoteck. Engr., Div., ASCE, 100, No. GT 12, 1300–1320.
- BOWDEN, F. P. and TABOR, D. (1964), The friction and lubrication of solids, Vol. 2, Clarendon, London.
- BYERLEE, J. D. (1967), Theory of friction based on brittle fracture, J. Appl. Phys. 38, 2928–2934.
- BYERLEE, J. D. (1970), The mechanics of stick-slip, Tectonophysics 9, 475-486.
- BYERLEE, J. D. and BRACE, W. F. (1968), Stick-slip, stable sliding, and earthquakes-effect of rock type, pressure, strain rate, and stiffness, J. Geophys. Res. 73, No. 18, 6031-6037.
- BYERLEE, J. and SUMMERS, R. (1976), A note on the effect of fault gouge thickness on fault stability, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 13, 35–36.
- DIETERICH, J. H. (1970), Time dependence in stick-slip sliding, Trans. Am. Geophys. Union 51, 423.

DIETERICH, J. H. (1972), Time-dependent friction in rocks, J. Geophys. Res. 77, 3690-3697.

- DIETERICH, J. H. (1975), A model for earthquake precursors based on premonitory fault slip, Trans. Am. Geophys. Union 56, 1959.
- DIETERICH, J. H. (1977), Preseismic slip and earthquake prediction, submitted to J. Geophys. Res.
- ENGELDER, J. T. (1974), Microscopic wear grooves on slickensides: Indicators of paleoseismicity, J. Geophys. Res. 79, 4387–4392.
- ENGELDER, J. T., LOGAN, J. M. and HAUDIN, J. (1975), The sliding characteristics of sandstone on quartz fault-gouge, Pure and Appl. Geophys. 113, 69-86.
- ENGELDER, J. T. and SCHOLZ, C. H. (1976), The role of asperity indentation and ploughing in rock friction: II. Influence of relative hardness and normal load, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr. 13, 155–163.
- HORN, H. M. and DEERE, D. N. (1962), Frictional characteristics of minerals, Geotechnique 12, 319-335.
- HOSKINS, E. R., JAEGER, J. C. and ROSENGREN, K. J. (1968), A medium-scale direct shear experiment, Int. J. Rock Mech. Min. Sci. 4, 143–154.
- JAEGER, J. C. and COOK, N. G. W. (1969), Fundamentals of rock mechanics, Methuen Co. Ltd., London.
- JAEGER, J. C. and ROSENGREN, K. J. (1969), Friction and sliding of joints, Australas. Inst. Mining Met. Proc. 229, 93–104.
- MACMILLAN, N. H., HUNTINGDON, R. D. and WESTWOOD, A. R. C. (1973), Relationship between ζ-potential and dislocation mobility, Martin Marietta Laboratories Technical Report 73-11c, 14 pp.
- MACMILLAN, N. H., HUNTINGDON, R. D. and WESTWOOD, A. R. C. (1974), Chemomechanical control of sliding friction in non-metals, Jour. Materials Science 9, 697–706.
- MARCUSON, W. F. and WAHLS, H. E. (1972), Time effects on dynamic shear modulus of clays, J. Solid Mech. and Foundations Div., ASCE 98, No. SM 12, 1359–1373.
- OHNAKA, M. (1973), Experimental studies of stick-slip and their application to the earthquake source mechanism, J. Phys. Earth, 21, 285, 303.
- RABINOWICZ, E. (1965), Friction and wear of materials, John Wiley, New York.
- SCHOLZ, C. H. and ENGELDER, J. T. (1976), The role of asperity indentation and ploughing in rock friction: I. Asperity creep and stick-slip, Int. J. Rock Mech. Men. and Geomech. Abstr. 13, 149–154.
- SCHOLZ, C. H., MOLNAR, P. and JOHNSON, T. (1972), Detailed studies of frictional sliding in granite and implications for earthquake mechanism, J. Geophys. Res. 77, 6392–6406.
- SWOLFS, J. S. (1971), Influence of pore-fluid chemistry and temperature on fracture of sandstone under confining pressure, Ph.D. dissertation, College Station, Texas A & M University, 64 pp.
- TEUFEL, L. W. (1976), The measurement of contact areas and temperature during frictional sliding of Tennessee sandstone, M.Sc. thesis, Texas A & M University, 65 pp.
- TEUFEL, L. W. and LOGAN, J. M. (1977), Effect of shortening rate on the real area of contact and temperatures generated during frictional sliding, Pure and Appl. Geophys., in press.
- TRUDEAU, P. T., WHITMAN, R. V. and CHRISTRIAN, J. T. (1974), Shear wave velocity and modulus of a marine clay, J. Boston Soc. Civil Engr. 61, No. 1, 12–25.
- WALKER, W. W. and DEMER, L. J. (1964), Effects of loading duration on indentation hardness, Trans. AIME, 230, No. 3, 613–614.

- WESTBROOK, J. H. and JORGENSEN, P. J. (1965), Indentation creep of solids, Trans. Amer. Inst. Min. Met. Engr. 233, 425-428.
- WESTBROOK, J. H. and JORGENSEN, P. J. (1968), Effects of water desorbtion on indentation microhardness anistrophy in minerals, Amer. Minerologist 53, 1899.
- WESTWOOD, A. R. C., GOLDHEIM, D. L. and LYE, R. G. (1967), *Rebinder effects in* MgO, Phil. Mag. 16, No. 141, 505-519
- WESTWOOD, A. R. C. and GOLDHEIM, D. L. (1968), Occurrence and mechanism of rebinder effects in CaF₂, Jour. Applied Physics 39, No. 7, 3401–3405.

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