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A note on contact stress and closure in models of rock joints and faults

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Abstract. We have re-examined asperity deformation predicted by joint closure models based on *Greenwood and Williamson* [1966] which use a statistical representation of loaded, rough surfaces. Although such models assume small elastic strains within contacting asperities (Hertzian contact) and well predict the observed dependence of closure on normal stress, large elastic normal strains measured in experiments violate the model assumptions. This inconsistency between observations and models can be resolved. The model dependence of closure on macroscopic normal stress results primarily from the statistics of the surface topography, and the functional dependence of closure on normal stress can be independent of assumed contact-scale elastic interactions. Thus, a joint model of the Greenwood and Williamson kind, modified to allow a portion of the elastic deformation to occur outside of the asperity contact region, predicts macroscopic behavior consistent with Hertzian models. Contact stresses derived from previously published models of this kind may be in error.

1. Introduction

Planar discontinuities such as faults and joints are important in determining the physical properties of the Earth's crust, especially where the confining stress is low. For example, rock elastic and poroelastic properties are affected by the presence of mismatched joints because of their high normal compliance [*Walsh and Grosenbaugh*, 1979; *Swan*, 1983; *Brown and Scholz*, 1985]. Electrical resistivity of dry rock depends on the percent of contact across joints and fractures [*Greenwood and Williamson*, 1966]. Fluid transport properties such as permeability and hydraulic diffusivity also depend on joint or fracture aperture, which can vary strongly with normal stress at shallow crustal conditions [*Gangi*, 1978]. As is the case for joints, the mechanical properties of faults, such as strength and sliding stability, are controlled by contact-scale interactions and deformation [e.g., *Dieterich*, 1978]. Because stress in the crust is generally limited by fault strength [see *Hickman*, 1991], and because some faults pose significant seismic hazard, models of fault strength and fault stability based on contact-scale interactions are of great interest in crustal and earthquake mechanics.

Existing models of the normal compliance of mismatched joints [*Walsh and Grosenbaugh*, 1979; *Swan*, 1983; *Brown and Scholz*, 1985], extensions of these for shear compliance [*Yoshioika and Scholz*, 1989], and some models of sliding friction [*Biegel et al.*, 1992; *Boitnott et al.*, 1992; *Wang and Scholz*, 1994] are derived from a statistical description of loaded rough surfaces proposed by *Greenwood and Williamson* [1966]. The Greenwood and Williamson model considers contact between a perfectly flat surface and another surface that is nominally flat but covered with fine-scale topography. When two surfaces are brought together to a separation of d (Figure 1a), the probability that any asperity on the nominally flat surface will make contact with the flat surface is

$$prob = \int_d^\infty \varphi(z)dz, \qquad (1)$$

where $\varphi(z)\,dz$ is the probability that a particular asperity has height z in the range $z+dz$. Assuming that all asperity contacts deform elastically and that the force F exerted by a single asperity is a function of the local deformation ($F=f(z-d)$), the total macroscopic resisting normal stress σ_n is

$$\sigma_n = \eta \int_d^\infty F(z-d)\varphi(z)dz, \qquad (2)$$

where η is the areal density of asperities (in units area^{-1}) on the surface. *Greenwood and Williamson* [1966] further assumed that the tips of all asperities are spherical and have constant radius of tip curvature β, thus requiring all contacts between the two surfaces to be circular. The relationship between the resisting force and the deformation of individual asperities is assumed to be given by Hertz's solution

$$F = \tfrac{4}{3}E'\sqrt{\beta}(z-d)^{3/2}, \qquad (3)$$

where E' is an elastic constant, the sum of $(1-\nu^2)/E$ for the two surfaces (ν is Poisson's ratio and E is Young's modulus) [see *Johnson, 1985*]. Use of (3) for F in (2) requires that the radius of each area of contact between the two surfaces is small compared with the height of the corresponding contacting asperity, and that the radius of each asperity contact is small with respect to the radius of curvature of the corresponding asperity tip [see *Johnson*, 1985]. Substituting (3) into (2) yields

$$\sigma_n = \tfrac{4}{3}\eta E'\sqrt{\beta}\int_d^\infty (z-d)^{3/2}\varphi(z)dz. \qquad (4)$$

Equation (4) is the foundation for subsequent work on the physical properties of joints and on the frictional strength of fault surfaces. Notably, *Brown and Scholz* [1985] have adapted (4) for the case of 2 nominally flat but rough surfaces in contact (Figure 1b). In this case, a composite surface profile, given by the sum of the two rough surface profiles, specifies the composite probability $\varphi(z)$. Although models of contact between two rough surfaces can be more complicated than (4), previous work has shown that in many cases (4) requires little modification. For example, although deformation resulting from non-normal incidence of asperities can lead to hysteresis in fracture closure [*Scholz and Hickman*, 1983], rigorous consideration of oblique contact suggests that local surface slope is small and tangential stress due to non-normal incidence can be ignored in deriving expressions for overall fracture compliance [*Brown and*

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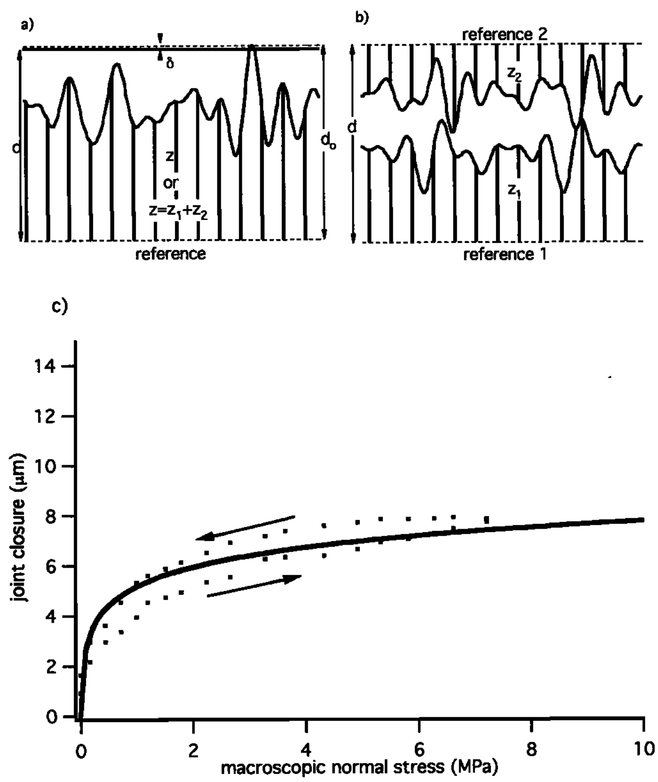

Figure 1. Representation of surface contact. a) Rough surface in contact with a flat [after *Greenwood and Williamson*, 1966]. The height of the rough surface z is measured with respect to a reference plane. The position of the flat d is also measured with respect to the reference. The macroscopic closure $\delta = d_0 - d$, where d_0 is the height of the tallest asperity above the reference. The deformation of an asperity in contact is z-d. b) Two rough surfaces in contact [after *Brown and Scholz*, 1985]. The actual topography is defined by upper surface heights z_2 measured with respect to the upper reference surface and lower surface heights z_1 measured with respect to the lower reference surface. In the model of *Brown and Scholz* [1985], the actual topography is replaced by a composite topography $z=z_1+z_2$, equivalent to the case shown in Figure 1a, and d_0, d and δ are as defined in a). c) Representative closure of unmatched joint surfaces (dots) [*Brown and Scholz*, 1985], where right- and left-facing arrows indicate loading and unloading curves, respectively. The solid line is a fit with the Greenwood and Williamson theory (4), assuming an exponential distribution of asperity heights $\varphi(z)$.

Scholz, 1985], at least for artificial joints (flat surfaces roughened by grinding). Similarly, contributions from variable asperity curvature are expected to be small. Study of the statistics of random surfaces suggests that under many circumstances β is approximately constant [*Nyak*, 1971]; in particular, for laboratory joints in rock there is no correlation between asperity tip curvature and height [*Brown and Scholz*, 1985]. Thus, equation (4) with the composite probability $\varphi(z)$ and $\sqrt{\beta}$ replaced with its average value $<\sqrt{\beta}>$ is adequate to describe the normal closure of many rock joints [*Brown and Scholz*, 1985]. Predictions of the Brown and Scholz model for the case where the distribution of asperity heights of the composite topography is exponential (i.e., $\varphi(z)=C_1\exp(-z/C_2)$) are roughly consistent with the observed dependence of closure on macroscopic normal stress [e.g. *Goodman*, 1976] (Figure 1c). More generally, when models of the Greenwood and Williamson kind are used to represent fractures or joint surfaces [*Walsh and Grosenbaugh*, 1979; *Swan*, 1983], the predicted dependence of elastic closure on normal stress matches experimental measurements well.

In this paper we show that when applied to actual laboratory data, a Greenwood and Williamson model which assumes Hertzian contact requires deformation on the contact scale that is too large. In particular, a portion of the deformation involved in joint closure must be accommodated outside of the region of asperity contact. Thus, despite the observation that such models well describe the laboratory measurements of joint closure under stress, there is a fundamental inconsistency between theoretical models and observations. We resolve this inconsistency.

2. Implied contact strain from topography and closure measurement

If the distribution of asperity heights $\varphi(z)$ in (4) is determined by surface profiling prior to experimental measurement of closure, then tests of the Greenwood and Williamson theory are possible [e.g., *Swan*, 1983; *Brown and Scholz*, 1985; *Yoshioka and Scholz*, 1989]. We are specifically interested in predicting average microscopic contact stress $<\sigma_c>$ as a function of macroscopic normal stress σ_n based on the mean value of asperity deformation $<\delta_a>$, e.g. the Hertzian contact stress $\langle\sigma_c\rangle = 0.42E_*\sqrt{\langle\delta_a\rangle/\langle\beta\rangle}$ [see *Johnson*, 1985]. However, while calculating values of contact stress based on published values of joint closure, we find large macroscopic closure that violates the assumptions of Hertzian contact. For example, Figure 2a shows a representative theoretical probability density function $\varphi(z)$ for asperities calculated according to the theory of *Nyak* [1971] using the parameters listed in Table 1 from surface profile measurements for a joint surface of fused silica [*Brown and Scholz*, 1985]. The range of asperity heights which experience some deformation z-d in a joint closure test on this sample (Figure 2a) range from height d_0 to d_0-δ_{max}, where d_0 is the surface separation at $\sigma_n=0$ and δ_{max} is the closure observed at the maximum applied normal stress of 7 MPa. Denoting the deformation experienced by a contacting asperity as $\delta_a=(z-d)$, we find the mean value of asperity deformation as a function of closure is

$$\langle\delta_a\rangle = \int_d^{d_0}(z-d)\varphi(z)dz \qquad (5)$$

(Figure 2b). Contacting asperities experience a typical deformation $<\delta_a>$ of 1.3 µm at 7 MPa maximum normal stress, corresponding to δ_{max}= 7.73 µm (Figure 2b and Table 2). Therefore, to satisfy the assumptions of Greenwood and Williamson theory, displacements of this magnitude must be accommodated elastically within the region of contact for the majority of contacts, and the tallest asperities must accommodate deformation approaching δ_{max}.

To determine if $<\delta_a>$ could be reasonably accommodated elastically we compare $<\delta_a>$ to the asperity tip radius of curvature because the ratio $<\delta_a>/\beta_{avg}$ is approximately the asperity strain. Figure 2c shows that β derived from surface profile measurements, for the same sample shown in Figures 2a and 2b, is approximately independent of asperity height. Thus, using the average tip radius of curvature β_{avg} = 9.1 µm we infer that asperity strains during this experiment on fused silica at 7 MPa macroscopic normal stress range are typically

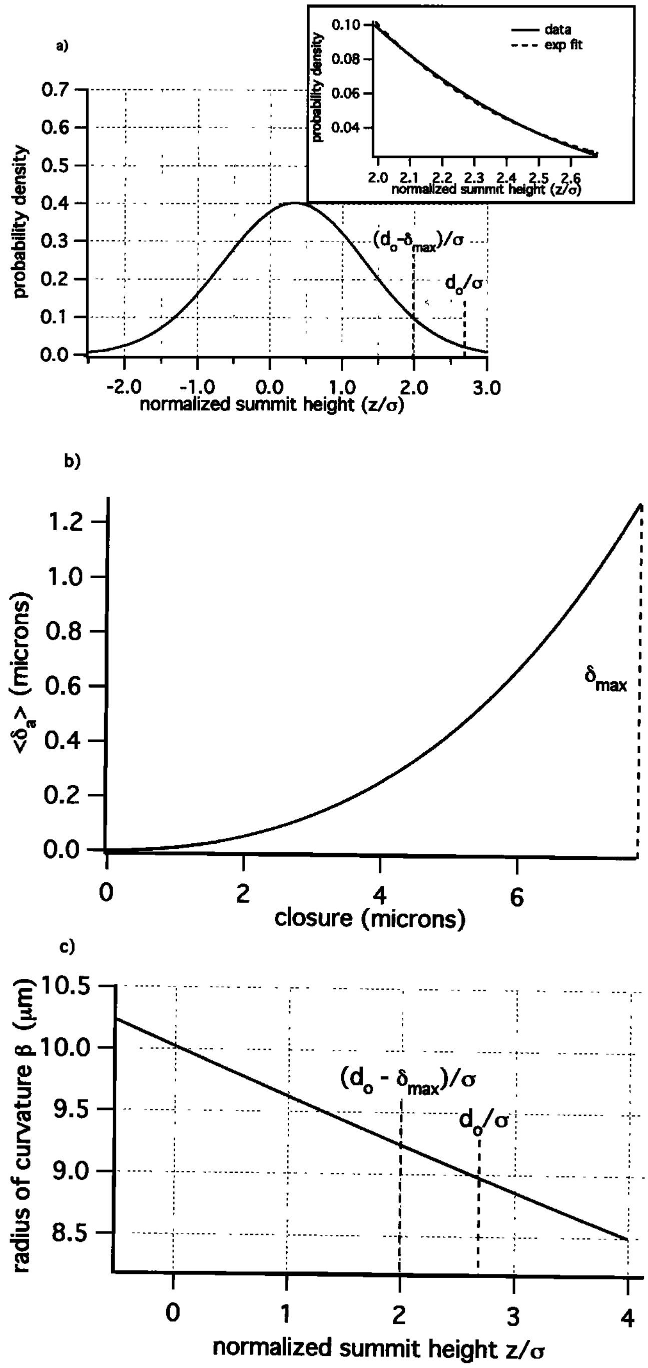

Figure 2. Properties of rough surfaces in contact. a) Probability density of asperity heights, calculated from profiles made by *Brown and Scholz* [1985]. Characteristics of the profile are listed in Table 1. Inset shows the range of asperity heights that are in contact at ~7 MPa normal stress, based on predictions of *Brown and Scholz* [1985]. b) The mean value of asperity deformation $<\delta_a>$ as a function of macroscopically measured closure δ calculated from equation (5) using data in Table 1. c) Distribution of asperity tip radius of curvature β calculated according to theory of *Nyak* [1971] from the profile data of *Brown and Scholz* [1985].

0.14 ($<\delta_a>$=1.3μm) and can be as high as 0.85 (δ_{max}=7.73μm). Results for Cheshire quartzite (Table 2, *Brown and Scholz*, 1986) indicate typical contact strains on the order of 0.31, with peak strains approaching 1.17. The strain implied by the measured elastic deformations represents a serious problem with this model because the deformation is too large to be accommodated elastically in quartzofeldspathic minerals or glasses. Similarly, contact spot radii calculated from Hertz's solution $\langle a\rangle = \sqrt{\langle\delta_a\rangle\beta_{avg}}$ [*Johnson*, 1985] are smaller but of the same order as β (Table 2) for the *Brown and Scholz* [1985; 1986] data, violating the conditions required for using equation (3).

Previous studies of quartzofeldspathic materials using such models have considered whether contact stresses reach the level of plastic yield [*Yoshioko*, 1994; *Brown and Scholz*, 1986]. To determine the threshold for plastic yield we evaluate the classical criterion of *Greenwood and Williamson* [1966] based on *Tabor* [1951], and an alternative criterion suggested by *Brown and Scholz* [1986]; $\delta_y = \beta(H/E')^2$ and $\delta_{yy} = 2\beta(H/E')^2$ are the classical and the modified criteria, respectively. H is the indentation hardness. Calculated values of these displacements for the *Brown and Scholz* [1985; 1986] fused silica and quartzite experiments (Table 2) indicate that the typical asperity should experience plastic strain because the critical displacement for yield is smaller than the typical asperity deformation $<\delta_a>$. In contrast, actual experimental measurements indicate that typical joint closures (ranging from 5 to 11 μm) are fully recoverable, therefore the deformation is almost totally elastic [*Swan*, 1983; *Brown and Scholz*, 1985; 1986]. Our conclusion is that the Greenwood and Williamson model with Hertzian contact is not consistent with observed elastic joint closure- it predicts contract strains well in excess of those required for yielding.

3. A non-Hertzian model

If Hertz's equation cannot be used with (2) to model asperity strain at low normal stress, why then does the Greenwood and Williamson model successfully describe the pressure dependence of joint closure (Figure 1c), and why are its extensions able to describe aspects of sliding friction [*Biegel et al.*, 1992; *Boitnott et al.*, 1992; *Wang and Scholz*, 1994]? We believe the answer is that the solution of (2) can have exactly the same form when other representations of asperity resisting force $F(z\text{-}d)$ are used.

This can be shown as follows. *Greenwood and Williamson* [1966] suggest that the range of asperity heights in contact at any given load can be well approximated by an exponential function $\varphi(z)\approx C_1 exp(-z/C_2)$, where C_1 and C_2 are constants. As shown in Figure 2a and elsewhere [*Walsh and Grosenbaugh*, 1979; *Swan*, 1983; *Power and Tullis*, 1992], the exponential distribution of contacting asperity heights is an adequate approximation for many natural and laboratory rock surfaces. In this case, equation (4) has the analytical solution

$$\sigma_n = C_1 C_2^{5/2}\eta E' \sqrt{\pi\beta}\exp^{-d/C_2} \qquad (7)$$

Table 1. Surface characteristics and closure model parameters [*Brown and Scholz*, 1985; 1986].

Exp.	σ (μm)	m_2	m_4 (μm^{-2})	d_0 (μm)	m_0 (μm^2)	α
CGL0394	11.2	0.112	4.53x10^{-3}	30.02	125.44	45.3
CCQ0073	12.9	0.111	5.11x10^{-3}	33.3	166.41	69.0

σ –standard deviation of asperity heights, calculated from surface profile measurements using theory of [*Nyak*, 1971], $\sigma = \sqrt{m_0}$

m_0, m_2, m_4 – the first 3 even moments of the power spectrum of a surface profile [*Nyak*, 1971; *Brown and Scholz*, 1985]

α – $m_0 m_4/(m_2)^2$ parameter describing the flatness of a power spectrum

d_0 – distance between reference planes at σ_n=0

Table 2. Measured maximum closure and calculated properties of typical asperity contacts [*Brown and Scholz*, 1985; 1986].

Exp.	δ_{max} (μm)	$<\delta_a>$ (μm)	β_{avg} (μm)	$<a>$ (μm)	δ_y (μm)	δ_{yy} (μm)
CGL0394	7.73	1.3	9.1	3.4	0.52*	1.03*
CCQ0073	10.43	2.8	8.9	5.0	0.57**	1.13**

* Fused silica E'=29.57 GPa [*Brown and Scholz*, 1985], H= 7 GPa [*McLellan and Shand*, 1984]

** quartzite E'= 39.7 GPa [*Brown and Scholz*, 1986], hardness assumed equal to that of quartz H= 10 GPa [*Evans*, 1984]

Noting that $d=d_0-\delta$ and defining a new constant $C_3 = C_1 C_2^{5/2} \eta E' \sqrt{\pi\beta}$, (7) can be rearranged to give

$$\delta = d_0 + C_2 \ln(\sigma_n / C_3), \qquad (8)$$

which is the same form as the empirical relationship commonly used to relate normal stress to joint closure [*Goodman*, 1976]. However, the functional dependence of closure on macroscopic normal stress in (8) does not require Hertzian contact. If, as assumed above, $\varphi(z)\approx C_1 exp(-z/C_2)$, and if the resisting force of an individual asperity has the general form $F=C_4(z-d)^p$, where p is an positive exponent and C_4 is a constant (c. f., equation 3), then the solution to (2) is

$$\sigma_n = \eta C_1 C_4 C_2^{(p+1)} \Gamma(p+1) \exp^{-d/C_2}, \qquad (9)$$

which differs from (7) by a constant. Here, Γ is the gamma function, for example having particular values $\Gamma(n)=(n-1)!$ and $\Gamma(n+1/2)=\sqrt{\pi}/2^n \cdot (2n-1)!!$, where n is a positive integer, ! is the factorial series, and !! is the double factorial series.

Having demonstrated that the experimental measurements of joint normal strain are elastic but too large to be accounted for by Hertzian contact, we conclude that some of the elastic strain is accommodated outside the region of contact. An alternative to (3) for the resisting force of an asperity is obtained by assuming that each asperity behaves as a linear elastic spring wherein the deformation is distributed over the entire height of the asperity instead of only in the contact region. Although the stiffness of each asperity might be expected to be inversely proportional to its height, the variation in height of asperities in contact during a joint closure test is typically very small (e.g. Figure 2a). As an approximation we therefore assume that the stiffness of all asperities in contact is independent of asperity height, i.e. $F=C_4(z-d)^1$ and (9) can be used. Thus, for our alternative model we obtain a solution for σ_n that has exactly the same functional dependence on d and η as the Hertzian model. We therefore argue that the macroscopic pressure dependence of closure predicted by models of the *Greenwood and Williamson* [1966] kind depends primarily on how many asperities are in contact and secondarily on the geometry and scale of the contact or asperity-scale elastic interactions, at least for artificial joints. In particular, if $\varphi(z)$ is an exponential and $F(z-d)$ is a power law, the form of the solution to (2) is entirely independent of assumptions about microscopic elastic interactions.

4. Conclusions

Elastic models of rock joints that use a statistical representation of surface roughness and assume Hertzian contact [*Greenwood and Williamson*, 1996] are invalidated by experimentally measured joint closure- the observed elastic displacements are too large. For artificial joints, the macroscopic pressure dependence of closure predicted by models of the Greenwood and Williamson kind depends primarily on how many asperities are in contact. By assuming that strains are accommodated within the entire height of contacting asperities, instead of in the immediate vicinity of contact, we obtain a model that can predict the same functional relationship between closure and pressure as predicted by Hertzian models. Our analysis suggests that microscopic contact stresses derived from previous models of rough surfaces in contact may be in error.

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